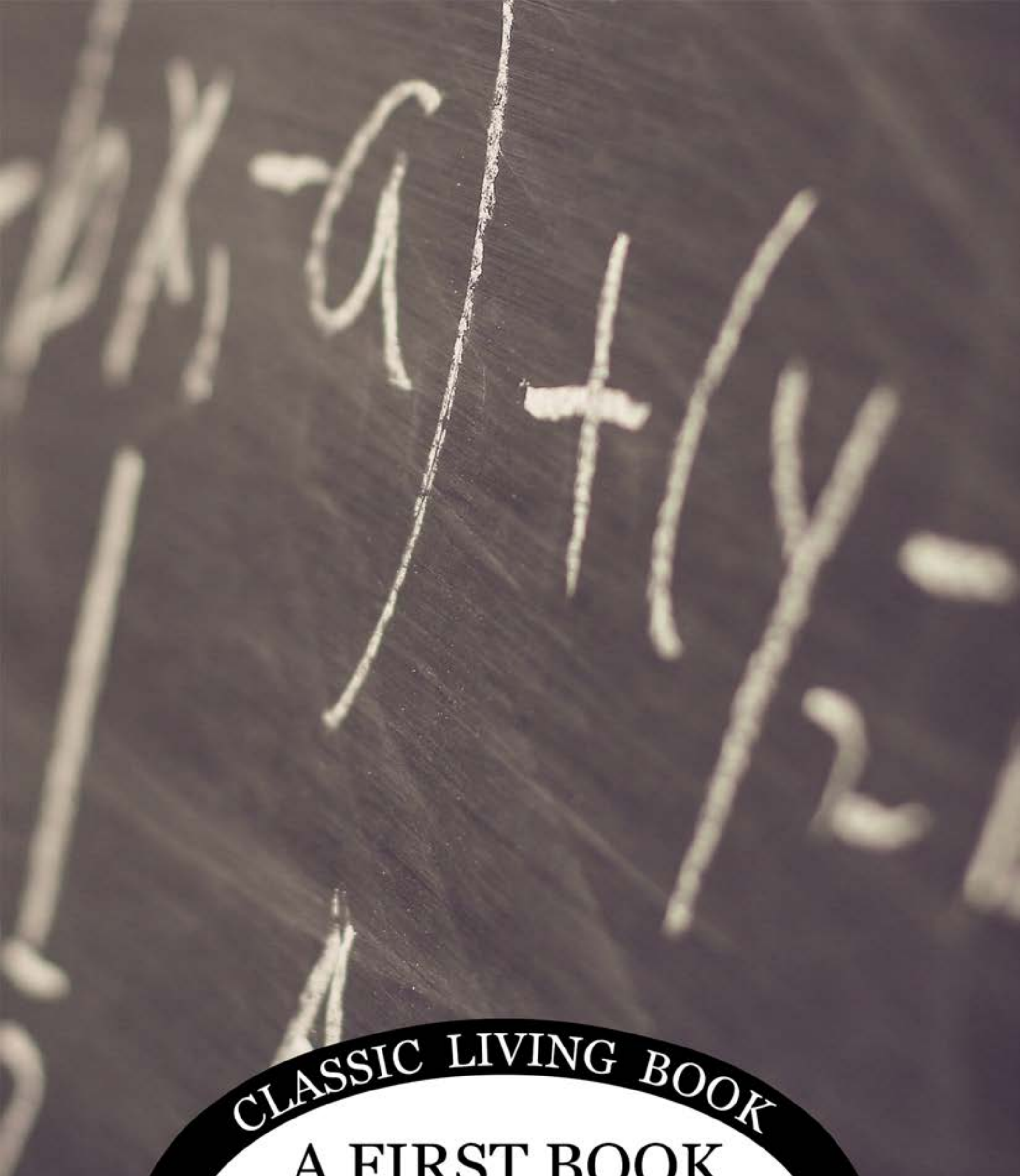

CLASSIC LIVING BOOK

A FIRST BOOK IN ALGEBRA

Fletcher Durell

COMPLETE AND UNABRIDGED

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A FIRST BOOK IN ALGEBRA

BY

FLETCHER DURRELL, Ph.D.

HEAD OF MATHEMATICAL DEPARTMENT,
THE LAWRENCEVILLE SCHOOL

AND

E. E. ARNOLD, M.A.

SUPERINTENDENT, THE PUBLIC SCHOOLS OF
THE PELHAMS, NEW YORK,
FORMERLY SPECIALIST IN MATHEMATICS, THE
UNIVERSITY OF THE STATE OF NEW YORK

PREFACE

THIS book has been written to meet the changes in High School work which have developed in recent years. For instance, most pupils now entering High School are younger and more immature than such pupils were even a decade ago. A majority of them also study only first year algebra and do not take the later more advanced course. Hence a demand has arisen that the first year in Algebra be simplified and made as directly practical as possible.

The outstanding features of this book are as follows:

I. *A simplification of subject matter.* Among the omissions are the involved use of the parenthesis, the larger part of factoring, complex fractions, special devices in the solution of equations, the method of comparison in solving simultaneous equations, the square root of complicated expressions, all treatment of fractional, negative, and literal exponents, and most of radicals.

II. *Increased and systematic use of the graph.* The omissions just specified make possible, among other things, an enlarged and systematized use of the graph. At the outset, the scales to be used on the axes and also partial diagrams for graphs are supplied. The other various stages by which the pupil proceeds from the simple to the more complex cases have been carefully developed. The pupil thus acquires an organized and comprehensive grasp and a power both to construct and to interpret graphs, giving a disciplinary training equal and perhaps superior to that conferred by the more technical algebraic topics omitted, and far exceeding them in vocational and cultural values.

III. *Increased use of the formula.* A like development has been made of the formula. In particular, almost every topic in arithmetic is stated, reviewed, and further developed by aid of the formula.

IV. *Improved treatment of written problems.* Particular attention is called to the new method of dealing with written problems, shown on pp. 4–7, and used throughout the book. Teachers agree that the written problem is the most difficult topic in algebra. It is believed that the method here presented meets this fundamental difficulty, just as the group method used in the authors' text on geometry has overcome the difficulty in teaching pupils to work originals in that subject.

V. *Large amount of oral exercise work in the use of algebraic language.* Each exercise in written problems is preceded by an oral exercise in the use of algebraic language. In certain of the examples of these sight drills, the more difficult written problems which follow in the next exercise are analyzed.

VI. *Organization of the chapters into Parts I and II.* In Part I of each chapter, the simple elements of the new topic are presented with as little theory as possible. It is intended that the class shall study in a first course or semester Part I of all the chapters in succession, and then return to the beginning of the book, and study the Parts II in order, reviewing each Part I, before taking up the Part II which follows it. The arrangement of material, however, is such that if the teacher prefers all the subject matter can be studied consecutively. It is strongly recommended, however, that the method of first studying all of the Parts I be followed.

It is generally understood that the most difficult problem in mathematical pedagogy is the transition from arithmetic to algebra. It is felt that the new departures in this book make a distinct contribution toward the solution of this problem.

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A FIRST BOOK IN ALGEBRA

CHAPTER I

SYMBOLS; FORMULAS; PROBLEMS

PART I

1. Some Algebra Which You Already Know. In arithmetic you have learned that an expression like "5 increased by 3" may be more briefly expressed as "5 + 3"; that is, + is a short way of writing "increased by" or "added to."

Similarly, the rule, "percentage is equal to the base multiplied by the rate expressed decimally," may be stated in a short way, thus $p = b \times r$.

Give two other instances from arithmetic where symbols are useful as short substitutes for words.

2. The Purpose of Algebra is to lessen work and obtain many other useful results by a greater use of symbols than is practiced in arithmetic.

3. Short Ways of Indicating Multiplication in Algebra. In algebra if a and b each stands for a number, a multiplied by b (or a times b) may be briefly expressed in any one of three different ways, viz.: $a \times b$, $a \cdot b$, or ab. That is, multiplication is sometimes expressed by the inclined cross, $\times$; sometimes by a dot; and sometimes by no symbol between a letter and its multiplier.

Thus, "3 times x" is written $3x$. Similarly, for $b \times r$, we may write br.

EXERCISE 1

1. In arithmetic "diminished by" is expressed by what symbol? In the short way write "7 diminished by 3."

2. In arithmetic "divided by" is expressed by what symbol? Write in a short way "12 divided by 4."

3. Write as long a list as you can of symbols which are used in arithmetic as short substitutes for words. Opposite each symbol write the word or words it stands for.

4. Find 6 per cent of 400. In this example which number is the base? The rate per cent? The percentage?

5. Count the letters that are used in writing the following rule, "The percentage is equal to the base multiplied by the rate per cent expressed decimally." Also count the number of letters and symbols in the formula $p = br$. Hence, in this case, the work of writing the rule is how many times as great as the work of writing the formula?

By the aid of symbols write the following in the shortest way you can:

6. Five plus seven.
7. 3 plus 4.
8. x increased by 4.
9. a plus b.
10. 7 minus 5.
11. y minus 5.
12. x minus y.
13. 3 times y.
14. x times y.
15. 3 times a times b.
16. x times y times z.
17. The product of b and r.
18. The product of i, r, and t.
19. x divided by 3.
20. The quotient of p and r.
21. a equals the product of l and w.
22. v equals the product of l, w, and h.
23. v divided by the product of l and w.

In reading $5x$, instead of reading "five times x," say simply "five x." In like manner, $5ab$ is read "five ab." Using this method when necessary, express in words:

24. $5+2$. **25.** $5+x$. **26.** $b+x$. **27.** $x-y$. **28.** $x \div y$. **29.** $7x$. **30.** xy. **31.** $\frac{ab}{p}$. **32.** prt. **33.** $3b+5xy$. **34.** $5+x-y$.

Find the value of each of the following when $a=1$, $b=2$, $c=3$, $x=4$, $y=6$:

35. $3+b$. **36.** $a+b$. **37.** $x-a$. **38.** $3x$. **39.** $\frac{x}{2}$. **40.** $\frac{x}{b}$. **41.** $\frac{bc}{x}$. **42.** $3ab$. **43.** abx. **44.** $\frac{x+y}{b}$. **45.** $2a+3ab$. **46.** $5x-y$. **47.** $\frac{2x+y}{b}$. **48.** $\frac{3y-4c}{x}$. **49.** $\sqrt{x}$. **50.** $\sqrt{bcy}$.

51. Find the value of lw when $l=8.5$ and $w=4$.

52. Find the value of br when $b=\$450$ and $r=.06$.

53. Find the value of $\frac{p}{b}$ when $p=60$ and $b=120$.

54. Find the value of $\frac{p}{r}$ when $p=36$ and $r=.05$.

55. If Mary has knit 6 garments and her older sister has knit 3 times as many, how many has her sister knit? How many have both knit together?

56. If Mary has knit x garments and her sister has knit 3 times as many, how many has her sister knit? How many have both knit together?

57. If Walter has earned twice as many dollars as his younger brother Harold, and Harold has earned x dollars, how many has Walter earned? How many dollars have both earned together?

4. Analysis of Problems.

Ex. 1. Mary knit a certain number of garments for the Red Cross and her older sister knit three times as many. Together they knit 48 garments. How many did each girl knit?

The problem may be analyzed and made ready for solution by the use of symbols, thus:

NUMBERS DEALT WITH	SYMBOLS FOR NUMBERS
Number of garments knit by Mary	$= x.$
Number of garments knit by her sister	$= 3x.$
Number of garments knit by both together	$= 48.$

EQUALITY FOUND AMONG THESE NUMBERS

Sum of the first two numbers = the last number.

Ex. 2. Walter and Harold made \$84 by gardening one summer. Walter, who was older and stronger, received a double share of the profits. How much did each receive?

NUMBERS DEALT WITH	SYMBOLS FOR NUMBERS
Number of dollars received by the boys together	$= 84.$
Number of dollars received by Walter	$= 2x.$
Number of dollars received by Harold	$= x.$

EQUALITY FOUND AMONG THESE NUMBERS

Sum of the last two numbers = the first number.

EXERCISE 2

Analyze each of the following problems in the same way in which the two problems which precede this exercise are analyzed. Keep your results for future use.

1. On a half holiday Robert caught a certain number of fish and his cousin Edward caught twice as many. Together they caught 36 fish. How many fish did each of the boys catch?

2. One day Ella picked a certain number of quarts of strawberries and her mother picked three times as many.

Together they picked 32 quarts. How many quarts did each of them pick?

3. Two boys together catch 84 fish. If the boy who owns the boat which they use receives twice as many fish as the other boy, how many fish does each boy receive?

4. A man left $12,000 to his son and daughter. To his daughter, who had taken care of him in his old age, he left a double share. What did each receive?

5. A man and boy by working a garden one summer made $128.80. If the man received a share of the profits three times as large as the share received by the boy, how much did each receive?

6. Two boys together gathered 1 bu. 4 qt. of hickory nuts. If the boy who climbed the trees received a double share, how many quarts did each receive?

7. Two girls made $18.60 by sewing. The girl who supplied the thread and machine received twice as much as the other girl. How much did each make?

8. Make up and work a similar example concerning two girls who kept a refreshment stand.

9. The total cotton crop of the world in a certain year was 15,000,000 bales, and the United States in that year produced three times as much as all the rest of the world. How many bales of cotton did the United States produce?

10. A farm is worked on shares. As the tenant supplies the tools and fertilizers, he receives twice as large a share of the profits as the owner of the farm. If the profits for one year are $6000, how much does the tenant receive? The owner?

11. If the sum of the areas of New York and Massachusetts is 57,400 sq. mi., and New York is 6 times as large as Massachusetts. what is the area of each state?

12. One number is 5 times as large as another and the sum of the numbers is 240. Find the numbers.

13. One number is twice as large as another and the sum of the numbers is 7.26. Find the numbers.

14. Separate $5\frac{1}{3}$ into two parts such that one part is 7 times as large as the other.

15. To look well, the middle part of a steeple should be twice as high as the lowest part, and the top part 8 times as high as the lowest part. If a steeple is to be 132 ft. high, how high should each part be?

16. A man wants to save $6000 in three years. If he is to save twice as much the second year as the first, and three times as much the third year as the first, how much must he save each year?

5. Solution of Problems. To illustrate the complete solution of a problem we shall now take Ex. 1, § 4, p. 4, repeat its analysis, and complete its solution.

Ex. Mary knit a certain number of garments and her older sister knit three times as many. Together they knit 48 garments. How many did each girl knit?

NUMBERS DEALT WITH	SYMBOLS FOR NUMBERS
Number of garments knit by Mary	$= x.$
Number of garments knit by her sister	$= 3x.$
Number of garments knit by both together	$= 48.$

EQUALITY FOUND AMONG THESE NUMBERS

Sum of the first two numbers = the last number.

Or, in symbols, $x + 3x = 48.$

Hence, $4x = 48.$

$x = 12$, *number of garments knit by Mary.*

$3x = 36$, *number of garments knit by her sister.*

CHECK. 36 is three times 12. Also $12 + 36 = 48.$

Hence all of the conditions of the given problem are satisfied by the numbers 12 and 36.

The teacher will note that the pupil in the work of searching out all the numbers that are contained in a given problem, and in arranging them in a list, is necessarily and perhaps unconsciously analyzing the problem, and therefore advancing a long way toward its solution. Hence the teacher is urged to insist that pupils follow the above form of solution in full.

EXERCISE 3

Analyze and solve the problems in Exercise 2.

Also analyze and solve each of the following problems:

1. A girl has $66 to spend for a hat, coat, and suit. She wants to spend twice as much for her coat as for her hat, and three times as much for her suit as for her hat. How much does she spend for each?

2. A man bequeathed $84,000 to his niece, daughter, and wife. If the daughter received twice as much as the niece, and the wife four times as much as the niece, how much did each receive?

3. A certain kind of concrete contains twice as much sand as cement and 5 times as much gravel as cement. How many cubic feet of each of these materials are there in 1000 cu. yd. of concrete?

4. Make up and work a similar example for yourself where the materials in the concrete are as 1, 2, 4.

5. In a certain kind of fertilizer the weight of the nitrate of soda equals that of the ground bone, and the weight of the potash is twice that of the ground bone. How many pounds of each material are there in a ton of fertilizer?

6. If the amount of potash in a given kind of glass is 5 times as great as the amount of lime, and the amount of sand 3 times as great as the amount of potash, how many pounds of each will there be in 4000 lb. of glass?

7. The railroad fare for two adults and a boy traveling for half fare was \$49.50. What was the fare for each person?

SUG. Let the smallest of the fares $= x$.

8. Separate 120 into three parts, such that the second part is twice as large as the first, and the third part three times as large as the first.

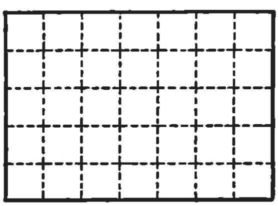
9. Separate 120 into three parts which are as 1, 2, 3.

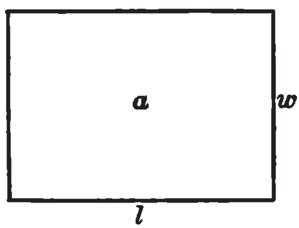
10. Of three numbers whose sum is 1800, the second is twice as large as the first, and the third is three times as large as the second. Find the numbers.

11. Of four girls who rolled bandages for the Red Cross, the second rolled as many as the first, and the third and fourth each rolled twice as many as the first. Together they rolled 480 bandages. How many did each roll?

6. A **formula** is a brief statement of a rule by the aid of symbols.

Ex. 1. Rule and formula for the area of a rectangle.





The number of square units in the area of the above rectangle $= 7 \times 5$, or 35. So in general

Rule for area of rectangle. (Number of square units in area) = (number of linear units in length) × (number of linear units in width).

Brief rule. Area = (length) × (width).

Formula for area of a rectangle.

$$a = lw.$$

Ex. 2. Rule and formula for percentage.

Rule. To find the percentage, multiply the base by the rate expressed decimally.

Formula. $p = br$.

EXERCISE 4

1. Construct a rectangle 8 inches long and 5 inches wide. Divide this rectangle into square inches, and find the area (area here is a number of square inches) of the rectangle by counting the number of square inches in it. Can you find the area by a shorter method?

2. In the formula $a = lw$, find the value of a when $l = 32$ and $w = 15$.

3. By the use of the formula $a = lw$, find the area of a rectangle whose length is 32 in. and whose width is 15 in.

4. In the formula $a = lw$, find the value of a when $l = 6.24$ and $w = 3.25$.

5. By use of the formula $a = lw$, find the area of a rectangle whose length is 6.24 in. and whose width is 3.25 in.

6. By use of the formula, find the number of square inches in the area of a rectangle whose length is 3 ft. 6 in. and whose width is 2 ft. 4 in.

7. Also find the number of square inches in the area of a rectangle whose length is 8 in. and whose width is p inches. Also in a rectangle whose length is r inches and whose width is s inches.

8. A boy's garden is 37 yd. long and 15 yd. wide. By use of the formula $a = lw$, find the area of the garden in square yards.

9. A certain floor is $23\frac{1}{2}$ ft. long and 12 ft. wide. By use of $a = lw$, find its area.

10. A concrete walk is 20 yd. long and 4 ft. wide. By use of $a = lw$, find its area in square yards.

11. A certain garden is to contain 140 sq. yd. If it is to be 20 yd. long, find how wide it must be. Use $a = lw$.

SUG. We have given $a = 140$, and $l = 20$, to find w. Substituting for a and l in the formula $a = lw$, we have:

$$140 = 20w.$$

Hence, $7 = w$,

or, $w = 7$. *Ans.*

12. A certain building lot is to contain 6000 sq. ft. If the lot is to be 250 ft. deep, how wide must it be? Use the formula $a = lw$.

13. A certain building lot is to contain a quarter of an acre and is to be 200 ft. deep. By use of $a = lw$, find how wide it must be. (An acre contains 43,560 sq. ft.)

14. On squared paper mark off a rectangle 12 spaces long and 7 spaces wide. Find the number of square units in the area of this rectangle by counting them. Also find the area by use of the formula. Compare the amount of work in these two methods of finding the area. (That is, estimate how many times as great the work of one process is as that of the other process.)

15. In the formula $p = br$, find p when $b = 420$ and $r = .05$.

16. By use of $p = br$, find 5 per cent of \$420.

17. By use of the formula $p = br$, find 12 per cent of 1650.

18. By use of $p = br$, find $37\frac{1}{2}$ per cent of 128. Also $62\frac{1}{2}$ per cent. Also $33\frac{1}{3}$ per cent. Also $87\frac{1}{2}$ per cent.

19. An agent sold a piece of property for \$3200 and received a commission of $2\frac{1}{2}$ per cent. By use of $p = br$, find how much he received.

20. A certain macadam road cost \$48,000. The county through which the road passed paid 40 per cent of the cost. By use of $p = br$, find how much money the county paid.

21. What per cent is 18 of 600?

SUG. We have given $p = 18$ and $b = 600$, to find r. Substituting for p and b in $p = br$, we have $18 = 600\,r$, etc.

22. If an agent received a commission of \$18 for selling a property for \$600, what per cent was his commission? Use the formula $p = br$.

23. A boy solves 16 out of 20 examples. What is his grade? What formula did you use?

24. If a baseball nine wins 27 games out of 35, what is its per cent of games won? Use $p = br$.

7. Algebraic Expressions. Any symbol or combination of symbols representing some number is called an algebraic expression.

Ex. $5\,x^2y - 6\,ab + 7\sqrt[3]{ax}$.

8. Terms. In order to treat algebraic expressions more efficiently it is convenient to regard the + and − signs in any given expression as separating the expression into terms.

Thus, $5\,a + 3\,b - 2\,c$ is considered as made up of the terms $5\,a$, $3\,b$, and $2\,c$.

9. Coefficients. In order to treat terms efficiently it is often convenient to treat part of a term separately and call it a coefficient.

Thus, in $5\,a$, 5 is called the coefficient of a. 5 shows how many times a is taken.

If the coefficient is 1, it is not written. Thus, instead of $1\,x$ we write x.

EXERCISE 5

1. How many terms are in the expression $5\,a + 2\,b$? Name these terms.

Treat in like manner:

2. $3\,x - 17\,y + 5\,z$. **3.** $3\,ab + 5\,bc - x + 2\,y$.