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PREFACE NUMBER ONE

JUST BETWEEN US, AND WORTH READING

These are the stories that were really told in the crisp autumn evenings, the Story-Teller sitting by the fire that burned in the great fireplace in the cottage by the sea. These are the stories as he told them to the Tease and the rest of the circle of friends known as the Crowd. Sitting by the fire and listening to the stories, in the lights and shadows of the dancing flames they could see the forms of Ching and Lugal and all the rest with their curious dress of long ago.

Night after night he told these tales of the ages past, stories unlike the make-believes they had often heard, stories of what might really have happened when the world was young, stories that the Crowd said were" different" because they told of much that was new, much that was curious, and much that was interesting.

So the Crowd learned many strange things that have happened in Number Land, but they learned much more than this; for the Story-Teller told them much that was interesting about the way in which boys and girls used to write in centuries long past — how Ching wrote on palm leaves, and Lugal on bricks, and Hippias on parchment. He also told them about many of the number puzzles that have delighted boys and girls for thousands of years, so that the Tease found new tricks to play on all her friends, and the Crowd found much to think about as the stories were related by the great log fire.

And you who read these stories should imagine yourselves sitting by the great log fire and listening to the Story-Teller. You should seem to see in the flames and the shadows the moving pictures of those who played their parts in Number Land when the world was learning as you do.

Is this history? Never mind. What is history but a story, and is not every story a history of something? Why bother our heads over history? For us the story is the important thing.

PREFACE NUMBER TWO

FOR THE GROWN-UPS, AND NOT WORTH READING

The story of our numbers, of the world's attempts to count, of the many experiments in writing numerals, and of the difficulties encountered through the ages in performing our everyday computations — all this is so interwoven with the history of humanity as to have an interest for every thinking person. As the world has grown, so the work with numbers has grown; when the world has faced the mysteries of the universe, numbers have assisted in solving its problems; when commerce and science have shown new needs in computation, arithmetic has always been ready to lend a hand. The history of mathematics is no small part of the history of civilization.

This being the case, it seems proper to relate at least some portion of the story of numbers to the pupils in our schools. It can be made quite as interesting as any other story of civilization, for it touches upon a subject with which the pupils in our schools are in daily contact, adding new values to the problems of arithmetic and giving a new perspective to the whole study of mathematics.

This book is intended for supplementary reading in the

elementary school. It is written in nontechnical language, and the effort has been made to connect with the history enough of the human element to make it more interesting than any mere recital of facts. With it there is also joined something of the history of writing materials, this being connected naturally with the story of our numbers. Chapters I-VIII can easily be read aloud, and the Question Box at the end of each chapter can be used as a basis for conversation or for written work.

The facts stated in the book are as nearly exact as the circumstances permit. It is not to be expected, however, that changes in the form of various numerals will be considered. Such changes are of no moment in a work of this nature and do not contradict the statement that the historical facts are presented with substantial accuracy.

It is the author's hope that this little series of human incidents will create a new interest not merely in the study of arithmetic but in the story of the development of our civilization.

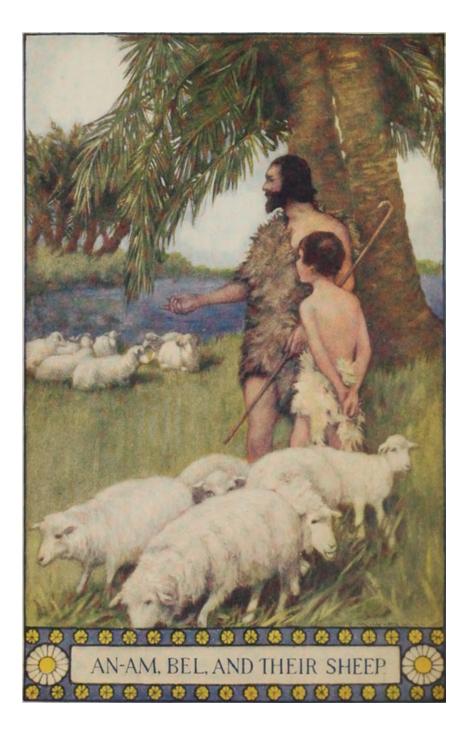
DAVID EUGENE SMITH

There is no knowledge that is not power. — EMERSON

There is no Past, so long as books shall live. — BULWER-LYTTON

They who lived in history only seemed to walk the earth again. — LONGFELLOW

Do not then train boys to learning by force and harshness; but direct them to it by what amuses their minds, so that you may be the better able to discover with accuracy the peculiar bent of the genius of each. — PLATO



CHAPTER I

HOW CHING AND AN-AM AND MENES COUNTED

The logs are burning in the great stone fireplace in the cottage by the sea. The Story-Teller sits in his easy-chair looking at a book of curious pictures and still more curious letters. She of the teasing ways is dancing through the open door, and with her are the others who make up what she calls the Crowd, tired with the hours of play upon the beach.

"Just one little story before bedtime," says the Tease.

"Just one," chime in the others.

"Not a single word," says he of the book with the curious pages.

"Oh, just one," says the Tease.

"Just one," begs the Crowd.

"Well, just one," says the Story-Teller, who knew all the time that he would submit. "Take your chairs, then, put a new log on the fire, and listen to the story of Ching and An-am (an am) and Menes (me'nez)."

The logs burned, the Crowd sat by the fire, and he of the curious book told this story:

It is so very, very long ago that not even the wisest men of China can tell the year or the century in which little Ching, the king's oldest son, played in the forests at the foot of Mount Yu, and painted a face on the shell of his biggest turtle, and told the soldier who guarded him what a lot of turtles he had. To be sure, Ching had only three turtles, but he didn't know a word for "three," and the soldier didn't, and not even the king could do more than say, "Yes, there are a lot of turtles."

For all this was so long ago that even in the oldest parts of the earth, of which China was one, most people could not count. It was before kings had palaces or crowns or royal robes, and when they were little more than savages. So we do not wonder that Ching, even though he was the son of a king, could only count "one, two," everything beyond that being a "lot." This was as far as people needed to count when Ching was playing in the forest at the foot of Mount Yu, for money was not invented, and we use our numbers to-day chiefly in buying the things we need. But in those days kings had many slaves and made them work, and sent them to kill animals, and made them bring back skins for clothing and meat for food. Few people needed to count, and only these few ever learned. Even the wisest men did not know much about the numbers that we use every day, because they had no need to do so.

At the time that Ching was growing up in China there lived on the plains of Mesopotamia (mes'o po ta'mi a), in southern Asia, a boy named An-am. He was the son of Bel, a shepherd of the country afterwards called the land of Babylon. Bel tended the sheep, and drove away the wolves that prowled about at night, and kept a careful watch to see that not one of his flock should wander away. One day Bel called out to An-am, "There are many sheep out there; drive them back." But really there were only a few sheep, for neither An-am nor Bel could count beyond three, and all larger numbers were called "many." Nevertheless Bel and An-am knew the sheep so well that they could tell if one was missing, just as a good shepherd dog to-day knows if one of his flock has gone astray. So An-am and Bel could count "one, two, three, many," and that was all they needed to know about arithmetic.

While Ching was playing in the forest at the foot of Mount Yu, and An-am was helping to watch the flocks that fed near the Euphrates (u fra'tez), another boy was living on the banks of the Nile in ancient Egypt. This boy's name was Menes, and he lived not far from the place where now the enormous dam holds back the waters of the great river. The little hut in which Menes lived was the grandest house that he or his father or his mother ever saw, and yet it had only a single room, and this was smaller than the schoolroom in which you study arithmetic.

For this was thousands of years ago, long before people had real houses, long before anyone knew how to read or write, long before the world had learned how to weave fine cloth, and long before men knew any other way to make a light than to rub two pieces of wood together until one of them was set on fire. Menes was proud of what he thought was the magnificent house in which he lived, although it was only a little hut, and he was glad to be able to say,

W e have a great many palm trees about our house," although there were only six. For Menes had heard his father and mother speak of one tree, of two trees, of three trees, and of four trees, but beyond that they simply said "a great many trees," for they had names for numbers only up to four, and all beyond that was a great many, just as we might speak of a great many apples.

When Ching and An-am and Menes grew to 'be men, and Ching became a king, and An-am became a manager of the Babylonian king's estates, and Menes became a great captain in the wars against the savages who lived in the south, Ching could only count to two, and An-am to three, and Menes to four, because this was as far as people in their countries could count in the days when the world was only just coming out of savagery, when money was unknown, and when no one measured land or buildings or the things which they traded with one another.

But many hundreds of years later other boys played in the forest at the foot of Mount Yu, and they counted "one, two, two and one, two twos, two twos and one, a lot." The world was growing, and people needed larger numbers, and so they counted as far as "two twos and one," which we call "five," and all beyond that was simply called a "lot."

And other boys helped to tend the flocks of Babylon, and their fathers taught them to count by threes, — "one, two, three, three and one, three and two, two threes, two threes and one, two threes and two, three threes, three threes and one, three threes and two, many," for they did not know a word for four, so they couldn't say "four threes," and they just said "many"; but of course they said another word, using the language of ancient Babylon. The world of Mesopotamia was growing older, and people needed more number names; but they still had no money, and a few such names were quite enough.

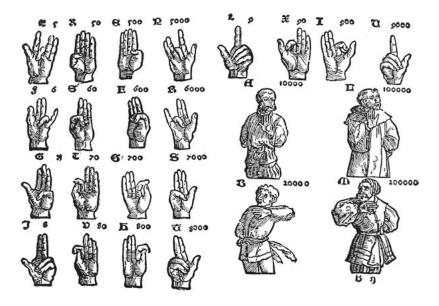
While the boys were counting to" two twos and one" in China and to "three threes and two" in Mesopotamia, Egyptian boys played under the palms where long before their time Menes had looked with pride upon his father's hut. No longer, however, was there just a hut with a single room, for the world was growing still older, and the descendants of the Menes of long ago had now a house with two rooms, and the Menes of this time had learned a new way of counting. The people along the Nile had found that the fingers of one hand would help them with their numbers, and so they made new names as far as five, and Menes now counted" one, two, three, four, five, five and one, five and two," and so on to "five fives and four"; and then he gave up and said "a great many." He could count farther than the Chings and the An-ams, but "five fives and four" is only twenty-nine, and this does not seem very far to us. But this was long before people could read and write, when they used knives made of stone, and when they thought the world was growing old, while to us it seems to have been very young.

Hundreds of years again went by, and still new Chings and An-ams and Meneses played in the forests of Yu, or on the plains of Mesopotamia, or on the banks of the Nile; but now the world began to feel that "five fives and four" was not large enough, even in ancient Egypt. Then it was that someone thought that if people could count to five on one hand, they might as well count to ten on two hands, and so the Ching and An-am and Menes of that day counted the trees and sheep by learning number names to ten, and then saying "one and ten, two and ten, three and ten," and so on to "ten tens, ten tens and one," and as much farther as they wished to go. The world had discovered that its ten fingers were useful in counting, and so it learned to count by tens; and this was one of the greatest discoveries that the world ever made. Although boys and girls speak different languages, they all have ten fingers, and so all civilized people to-day count by tens.

Near the equator, where the climate is hot, and where people did not wear shoes, they counted their toes as well as their fingers, learning separate number names to twenty, — not "one and ten," "two and ten," and "three-ten" (thir-teen), but "eleven," "twelve," and so on, with special names, to twenty, which they sometimes called "man finished," and beyond that they counted by twenties. Some of these people wandered to other countries and carried along with them their way of counting. But most of the people of the world did as the children of Ching and An-am and Menes did, — they counted by tens. When we hear of "three score years and ten," and when the French speak of "four twenties" instead of eighty, we have two remaining bits of the old way of counting by twenties.

Thus the world learned from Ching and An-am and

Menes, and from their children and their children's children, and so on for hundreds and hundreds of years, first to count by twos or threes, and then by fives, and then by tens, and sometimes by twenties. A few people tried to count by twelves, and so we have twelve inches in a foot, twelve ounces in the ancient pound, and twelve things in a dozen, but the reason why the world came to count by tens was because Ching and An-am and Menes and you and I have just ten fingers on our two hands.



REPRESENTING NUMBERS BY THE HAND

From a book printed nearly four hundred years ago. It shows the way in which numbers were represented by the hand.

Long after the early days of which we have been speaking, the world learned how to write numbers. Because different races wrote them in different ways, the merchants who traded with others whose language they did not speak represented numbers by their fingers. For at least two thousand years the merchants of different countries made number signs with their fingers in bargaining at the great fairs where they met to buy and sell the goods that thus went from country to country, — spices from India, silks from the land of Ching, wool from the ancient home of An-am, and dates from the palm trees under which Menes played many centuries before.

"Did they really count like this?" asked the Tease.

"Really," replied he of the curious book.

"I think it is funny," said the Tease.

"No funnier than your way would seem to Ching," said the Story-Teller.

"Do we have another story to-morrow night?" asked one of the Crowd.

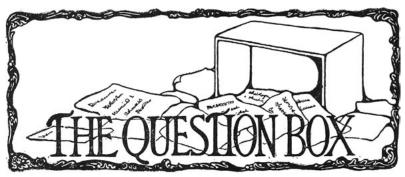
"Not another story ever," replied the Story-Teller, "unless you go to bed."

"And then?"

"To-morrow we must all fill a question box, and each must answer every question."

"And then?" asked the Tease.

"And then? Well, then we shall see," replied the Story-Teller.



1. How far could the first Ching mentioned in the story count?

2. Why did Ching not learn to count as we do?

3. What need have we for counting that Ching did not have?

4. Where did Ching live?

5. How far could the first An-am count?

6. If An-am could count no farther, how could he tell if one sheep was missing out of twenty sheep?

7. Why was it unnecessary for An-am to count farther?

8. Where did An-am live?

9. What name did the first Menes give for numbers beyond four?

10. Where did Menes live?

11. How did people come to count by tens?

12. By what other numbers than ten did the world sometimes count?

13. Why did people use one of the numbers referred to in Question 12?

14. What practical use did the world make of representing numbers on the fingers?