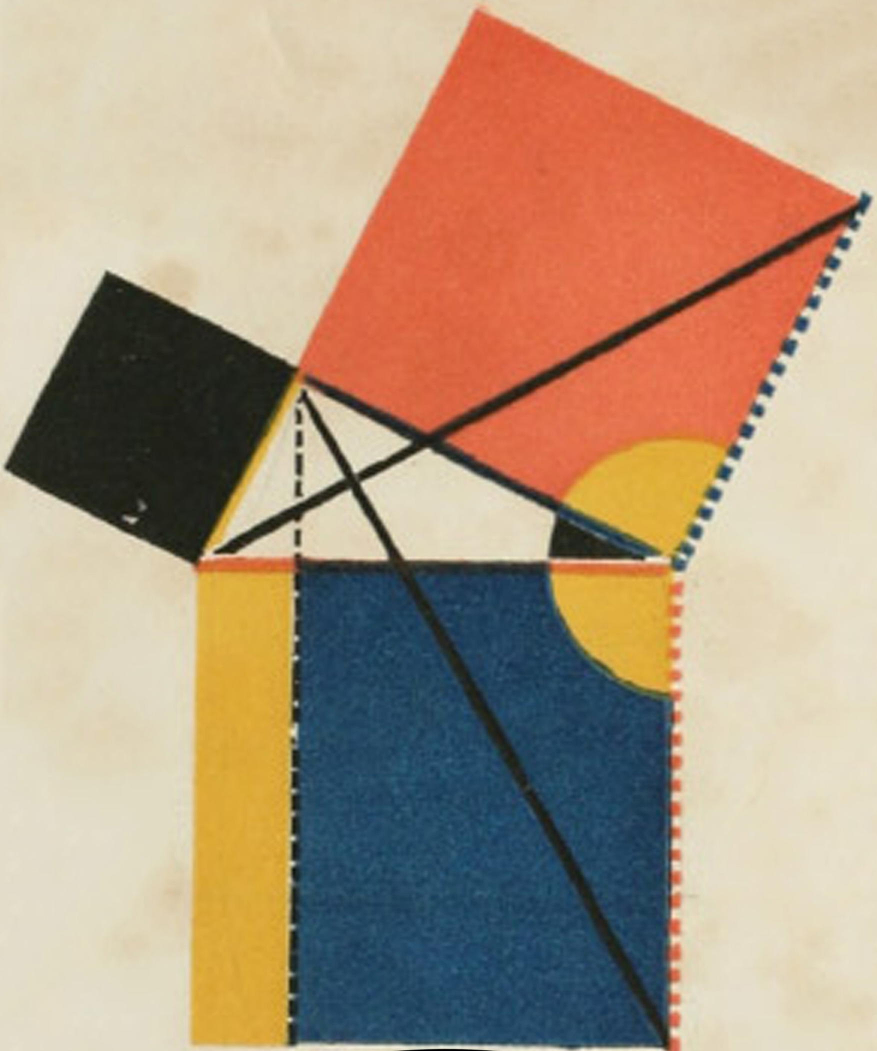

CLASSIC LIVING BOOK

PLANE GEOMETRY

Fletcher Durrell

COMPLETE AND UNABRIDGED

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PLANE GEOMETERY

WITH ANSWER KEY

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PREFACE

THE main object in writing the present book has been to produce a text in plane geometry which the average high school class can be fairly asked to cover in one year; which omits none of the essential theorems and, at the same time, embodies certain important improvements in the teaching of the subject; and which initiates the student into the subject in a natural manner, and arouses and sustains his interest in the facts and applications of geometry.

The outstanding features of the book are as follows:

I. *A reduction in subject matter.* The list of propositions used corresponds closely to the Harvard Syllabus, and the number of propositions has been reduced by about one fourth, as compared with those customarily included. The book, moreover, taken as a whole, covers the recommendations of the Committee of Fifteen of the National Education Association and hence exactly meets the requirements of the College Entrance Examination Board.

II. *Improvements in the organization of material.* In this respect, special attention is called to

1. The introduction, which furnishes a natural approach to geometry by informal treatment and by use of facts known to the student.

2. The construction work in exercise groups 1–6. These exercises familiarize the pupil with the use and value of the straightedge, compasses, and protractor, and give him a self-active interest in the subject.

3. The simplified, pedagogical order of arrangement of the first fifteen propositions of Book One, which have special value as an introduction to demonstrative work in geometry. No

one of these propositions should prove a stumbling block to the average pupil. The authors are confident that the arrangement of propositions in the remainder of the book will be found equally satisfactory.

4. The arrangement of the proofs as steps and reasons in parallel columns. This is a help in cultivating the logical faculty in the pupil and is an important aid to the teacher in inspecting and correcting written work.

5. The steps in proofs for which pupils must supply reasons. This aims to make the pupil independent of the book, fosters a spirit of original thinking, and develops mathematical intuition.

6. The theory of limits has been used only in informal ways in accordance with the recommendations of the Committee of Fifteen.

7. Every construction figure contains all the necessary construction lines.

III. *Improvements in the methods of teaching pupils to solve original exercises.*

1. The most valuable of these is the improved method of analysis, presented in § 173 (p. 90). This method is so stated that it may be utilized much earlier if the teacher wishes.

2. The group method of solving originals, which formed so successful a feature of Dr. Durell's earlier Geometry, has been retained and improved, as by the insertion of introductory and simpler groups throughout the text.

IV. *The development of practical applications and of efficiency values of geometry.*

1. The treatment of the practical applications has been simplified. Whenever the correlation is close and when these applications clarify or promote interest, they have been inserted in the text in such a way as to give them the maximum effect.

2. Emphasis is placed on the efficiency values of theorems and principles.

As a whole, the object of the book has been to make the teaching, study, and later use of geometry as efficient as possible in relation to present conditions.

CONTENTS

PAGE

Definitions and First Principles 7

Book I. Rectilinear Figures 29

Book II. The Circle 109

Book III. Proportion; Similar Polygons 178

Book IV. Areas of Polygons 223

Book V. Regular Polygons; Measurement of the Circle; Maxima and Minima; Symmetry 247

History of Geometry 275

Methods of Numerical Computation 282

Review Exercises 284

Appendix 294

Formulas of Plane Geometry 311

SYMBOLS AND ABBREVIATIONS

$+$ *plus*, or *increased by*.
$-$ *minus*, or *diminished by*.
$\times$ *multiplied by*.
$\div$ *divided by*.
$=$ *equals; is* (or *are*) *equal to*.
$\stackrel{m}{=}$ *is measured by*.
$>$ *is* (or *are*) *greater than*.
$<$ *is* (or *are*) *less than*.
$\sim$ *is* (or *are*) *similar to*.
$\therefore$ *therefore*.
$\perp$ *perpendicular*, *perpendicular to*, or *is perpendicular to*.
$\perp$s *perpendiculars*.
$\parallel$ *parallel*, or *is parallel to*.
$\angle$, $\angle$s *angle*, *angles*.
$\triangle$, $\triangle$s *triangle*, *triangles*.
▱, ▱s *parallelogram*, *parallelograms*.
$\odot$, Ⓢ *circle*, *circles*.
$\frown$ (as in $\overset{\frown}{AB}$) *arc*.

Adj., *adjacent*.
Alt., *alternate*.
Ax., *axiom*.
Comp., *complement*.
Constr., *construction*.
Cor., *corollary*.
Corr., *corresponding*.
Def., *definition*.
Ex., *exercise*, or *example*.
Ext., *exterior*.
Fig., *figure*.
Geom., *geometry*.
Hyp., *hypothesis*.
Ident., *identity*.
Ineq., *inequality*.
Int., *interior*.
Opp., *opposite*.
Post., *postulate*.
Prop., *proposition*.
Rt., *right*.
St., *straight*.
Sug., *suggestion*.
Sup., *supplement*.

Q.E.D. *quod erat demonstrandum;* that is, which was to be proved.
Q.E.F. *quod erat faciendum;* that is, which was to be made.

A few other abbreviations and symbols will be introduced and explained later.

PLANE GEOMETRY

DEFINITIONS AND FIRST PRINCIPLES

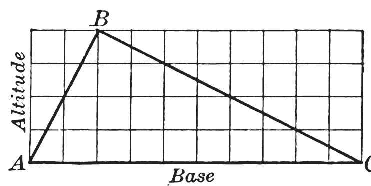
1. Some geometry which you already know. — From your study of arithmetic, and in other ways, you have already learned some of the most important properties of the straight line, circle, triangle, cube, sphere, and some other similar objects.

You know that a solid has three dimensions, viz.: length, breadth, and thickness. How many dimensions has a surface? A line? A point?

2. Efficient methods of treating geometric objects. — In dealing with objects like those just named in § 1, you have learned that often it is an advantage to use certain methods rather than others.

EXERCISES: GROUP 1

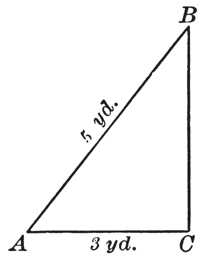
Ex. 1. Obtain the area of the triangle *ABC* by counting the small squares in the triangle (piecing together parts of squares). Also obtain the same area by multiplying the number of linear spaces in the base by the number in the altitude, and taking one half the product. Estimate how many times as much work you did in the first process as in the second.



Ex. 2. The number of bushels of wheat in a given bin might be determined by filling a bushel measure with wheat from the bin, time after time, till the wheat is exhausted, and counting the number of times the bushel measure is used. The number of bushels might be determined also by measuring (in feet) the three dimensions of the

bin and dividing their product by the number of cubic feet in a bushel ($1\frac{1}{4}$ cu. ft. approx. = 1 bu.). Compare the amount of work in the two processes, assuming that the bin is a large one.

Ex. 3. A given ladder AB is 5 yd. long, and its foot A is 3 yd. distant from the wall BC. In order to determine BC, which is easier: to measure BC, or to use the following computation?



No. yd. in $BC = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = 4.$

For the following exercises the student should have a ruler with an edge divided into inches and eighths of an inch.

Ex. 4. By folding a piece of paper, form a straight edge. With the aid of this straight edge, draw four lines of unequal length. Estimate the length of each of these lines and then measure the length of each to the nearest $\frac{1}{8}$ inch. Tabulate your results as follows:

LINE	ESTIMATED LENGTH	MEASURED LENGTH	ERROR
1			
2			
3			
4			

Ex. 5. With the paper straight edge, draw a line which you estimate to be twice as long as line 1. Check your estimate by measuring with the ruler the length of the line drawn.

Ex. 6. With the paper straight edge, draw a line and by estimate mark the middle point of the line. Test your estimate by use of your ruler.

Ex. 7. With the straight edge, draw a line which you estimate to be equal to the sum of lines 1 and 2. Check your estimate by use of the ruler.

Ex. 8. With the straight edge, draw a line which you estimate to be equal to the difference between lines 2 and 3. Check by use of the ruler.

Ex. 9. With the straight edge, draw a line and by estimate mark points which divide the line into three equal parts. Check with the ruler.

Ex. 10. In each of the following figures estimate whether the line *a* is longer or shorter than the line *b*. Check your estimate with the ruler.

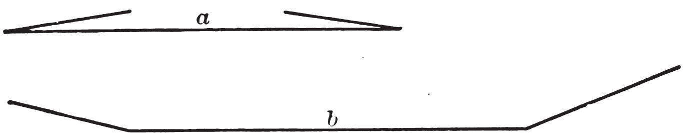


FIG. 1

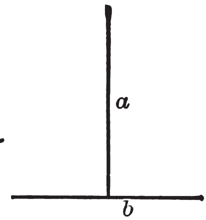


FIG. 2

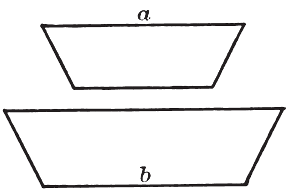


FIG. 3

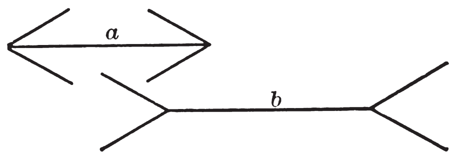


FIG. 4

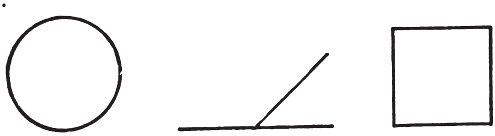
Ex. 11. Of the geometric objects — the solid, surface, line, and point — which is the boundary of a solid? Of a surface? Of a line?

3. **Geometry** is the study of the most efficient methods of dealing with the shape, size, and position of objects.

4. The **fundamental geometric objects** are the point, line, surface, and solid. For the present, you understand sufficiently well what these are without studying formal definitions of them.

5. A **geometric figure** is a point, line, surface, or solid, or any combination of these.

Thus, in arithmetic you have already used geometric figures like the following:



6. The **form** or **shape** of a figure is determined by the relative position of the various parts of the figure.

7. Similar geometric figures are those which have the same *shape*.

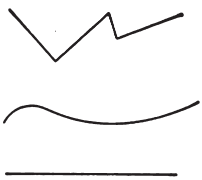
Equivalent figures are those which have the same *size*.

Equal or **congruent figures** are those which have the same *shape* and *size*, and can, therefore, be made to coincide.

8. A **point** is represented to the eye by a dot and is named by a letter affixed to the dot; as ·A, called the point A.

LINES

9. Straight line. — You already know that a straight line is the shortest line that can be drawn joining two points, and are familiar with some of the other useful properties of a straight line.



The word "line" is often used instead of "straight line."

10. A **curved line** is a line no portion of which is straight.

The word "curve" is often used for "curved line."

11. A **broken line** is a line made up of different straight lines.

EXERCISES: GROUP 2

Ex. 1. On a sheet of paper or on the blackboard, locate a point and draw two lines through this point.

Ex. 2. Can the lines drawn in Ex. 1 meet at another point?

Ex. 3. Can more than two lines be drawn through the given point? How many?

Ex. 4. On a flat surface can you draw two lines which would not meet, however far the lines are produced?

Ex. 5. On a flat surface, if you can, draw three lines, no two of which will meet on being produced.

Ex. 6. On a flat surface, how many lines can be drawn so that they will not meet?

Ex. 7. If a line is drawn on the given surface and meeting one of the lines in Ex. 6, how many of the other lines will it meet if it is extended?

Ex. 8. On a sheet of paper or on the blackboard, locate two points and draw a line passing through both of these points.

Ex. 9. Can another line be drawn passing through the two points of Ex. 8?

Ex. 10. Can more than one curved line be drawn passing through these two points?

Ex. 11. How many points are required to fix the position of a line?

Ex. 12. On a piece of paper or on the blackboard, draw a line along the edge of your ruler; then turn the ruler over and fit the same edge to the line. Will this test the edge of the ruler for straightness?

Ex. 13. Fold a sheet of paper. Test the edge of the fold in the same way that you tested the edge of the ruler in Ex. 12.

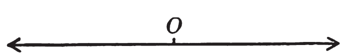
Ex. 14. Fit the edge of your ruler to the edge made by the fold of the paper (Ex. 13). How many points in the two edges must be made to coincide, in order that the two edges shall coincide throughout?

Ex. 15. Why are two sights necessary for a gun?

Ex. 16. In order to hold a straight iron rod in a given position, at how many points is it necessary to fasten the rod rigidly?

Ex. 17. When a farmer wishes to set out three or more apple trees in a straight row, how does he proceed?

Ex. 18. From a point on a line which is not an end point (as the point O), how many directions are indicated by a line?



Ex. 19. If one of these directions is east, what is the other direction?

Ex. 20. Point out two intersecting lines in the room.

Ex. 21. Point out two lines which would not meet on being produced.

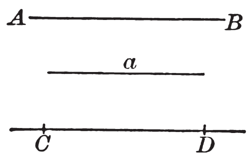
Ex. 22. Which of the capital letters of the alphabet are formed by straight lines? Curved lines? Broken lines? Curved and straight lines combined?

12. A **rectilinear figure** is a figure composed only of straight lines. A **curvilinear figure** is a figure composed only of curved lines. A **mixtilinear figure** is a figure containing both straight and curved lines.

13. Kinds of straight line. — A straight line may be definite or indefinite in length.

The line of definite length is sometimes termed **a segment** or **sect.**

14. Naming a straight line. — A straight line is named by naming two of its points; as the line *AB* (a sect), or the line *CD* (indefinite in length). A segment or sect may also be denoted by a single letter, usually small; as the line *a*.

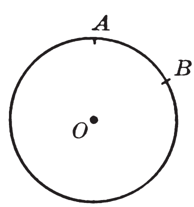


15. Parallel lines are lines in a flat surface which will not meet however far they are produced; as the lines *PQ* and *RS*.

16. A **circle** is a closed curve all points of which lie in the same flat surface and are equidistant from a point called the **center.**

An **arc** is any portion of a circle; as *AB*.

A **radius** of a circle is a line drawn from the center to any point on the circle.



17. Compasses are used to draw circles, and also to mark off and compare segments of lines.

EXERCISES: GROUP 3

Ex. 1. Draw a line and denote it by l. Also draw two much shorter lines and denote them by m and p, respectively.

Ex. 2. By use of the compasses, on l mark off a part equal to m.

Ex. 3. Construct a line equal to $l - 2p$. To $l - m - p$.

Ex. 4. Construct a line equal to $l + m - p$. To $l + p - 2m$.

Ex. 5. Draw a line and on it mark off three equal segments in succession.

Ex. 6. Draw a line three times as long as a given line.

Ex. 7. Draw a circle with a radius of three fourths of an inch.

Ex. 8. Draw two circles having the same center but different radii.

Ex. 9. Draw two circles which intersect.

Ex. 10. Draw two circles which do not intersect.

Ex. 11. Draw a line, and with each end of the line as a center draw circles which will intersect.

Ex. 12. In Fig. 1, does one of the circles *a*, *b*, *c*, appear to you to

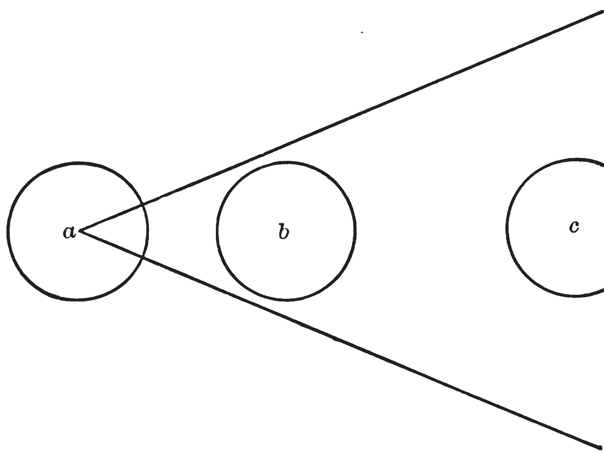


FIG. 1

be larger than the other two? Does one of them appear to be smaller than the others? Determine the relative size of the three circles by use of the compasses.

Ex. 13. In Fig. 2, which half circle has the greater apparent radius? Determine the relative size of the two half circles by use of the compasses.

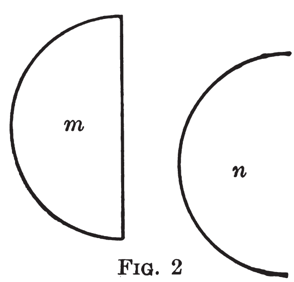


FIG. 2

Ex. 14. By use of the ruler and compasses, copy the following figures:

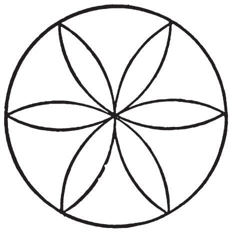


FIG. 3

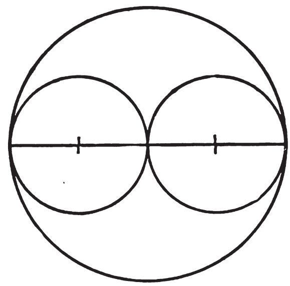


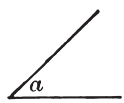
FIG. 4

ANGLES

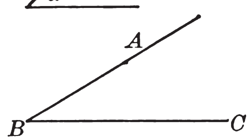
18. An **angle** is the amount of opening between two straight lines which meet at a point.

The **sides** of an angle are the lines whose intersection forms the angle. The **vertex** of an angle is the point in which the sides intersect.

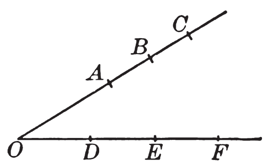
19. Naming an angle. — (1) The most convenient way of naming an angle is to place a letter or figure inside the angle and near the vertex; as the angle *a*.



(2) The most precise way is to use three letters: one for a point on each side of the angle, with the letter at the vertex between these two letters; as the angle *ABC*.



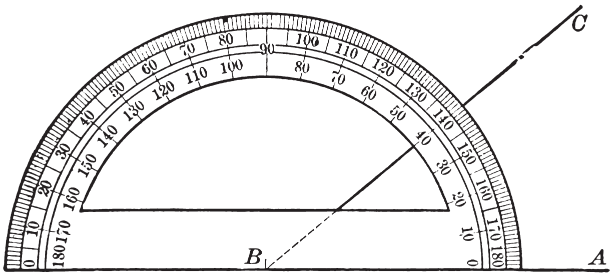
Since the size of an angle is independent of the length of its sides, the points named on its sides may be taken at any place on its sides. Thus the angles *AOD*, *BOD*, *BOE*, *AOF* are all the same angle.



(3) In case there is but one angle at a given vertex, the letter at the vertex alone may be used to denote the angle; as the angle *O* in the last figure.

20. Unit of angle. — The customary unit of angle is the **degree** (°). If a circle is divided into 360 equal parts, and the ends of one of these parts are joined with the center by two straight lines, the angle formed at the center is 1°.

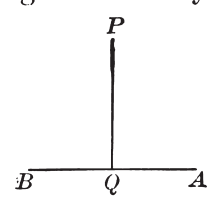
21. A **protractor** is a convenient instrument for measuring angles. It is a half circle with its rim divided into 180 equal parts, called **degrees of arc**. The center of the half circle is indicated by a dot or dash (see the mark at B).



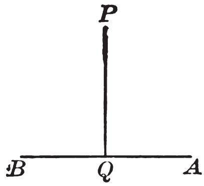
To measure an angle ABC, place the protractor over the angle so that the center of the protractor is directly over the vertex of the angle and the zero mark on the scale is over one side (or a side produced) of the angle, as BA. The point where the other side, BC, of the angle ABC crosses the scale indicates the number of degrees in the angle. (On the diagram this angle is 40°.)

22. A **straight angle** is an angle whose sides lie in the same straight line and extend in opposite directions from the vertex; as the angle AOB.

23. A **right angle** is one of two equal angles made by one straight line meeting another straight line. Thus, if the line PQ meets line AB so as to make angle PQA equal to angle PQB, each of these angles is a right angle. A right angle is half of a straight angle.



24. A **perpendicular** is a line that makes a right angle with a given line. Thus PQ in the figure is perpendicular to BA.

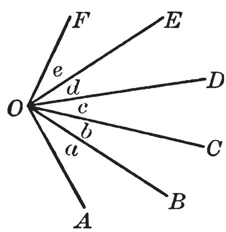


The **foot** of a perpendicular is the point in which the perpendicular meets the line to which it is drawn. Thus Q is the foot of the perpendicular PQ.

EXERCISES: GROUP 4

Ex. 1. Draw two lines that intersect. Measure the four angles formed by the lines. Find the sum of any two of the angles which are adjacent. Find the sum of all four of the angles.

Ex. 2. Draw a figure similar to the adjoining one, but having longer sides to the angles. Measure each of the angles a, b, c, d, and e.



Ex. 3. Compute the number of degrees in $\angle AOD$. In $\angle COF$. In $\angle AOE$.

Ex. 4. Verify your answers to Ex. 3 by measuring the angles named in it.

Ex. 5. By use of the protractor, construct angles of 30°, 60°, 90°, 15°, 45°, 120°, 135°, 180°.

Ex. 6. From a point P in a given line RS, draw a line PM making the $\angle MPS$ equal to 50°. Also measure $\angle MPR$. Find the sum of the two angles.

Ex. 7. On the diagram of Ex. 6, draw PN making $\angle NPR$ equal to 90°. Measure $\angle NPS$.

Ex. 8. Draw an angle of 60°. How many points on the scale of the protractor between the sides of this angle are marked 30°? Hence, how many bisectors can an angle of 60° have?

Ex. 9. How many lines can be drawn which will bisect any given angle?

Ex. 10. Bisect the angles MPS and MPR in Ex. 6. Measure the angle formed by the two lines which bisect these angles.

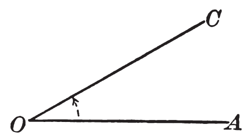
Ex. 11. Draw any two angles and then construct an angle equal to their sum.

Ex. 12. Draw two unequal angles and then construct an angle equal to their difference.

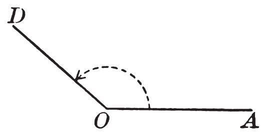
Ex. 13. How many points on the scale of the protractor are marked 90°?

Ex. 14. How many lines can be drawn perpendicular to a given line at the same point?

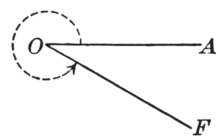
25. An **acute angle** is an angle less than a right angle; as the angle *AOC*.



26. An **obtuse angle** is an angle greater than a right angle but less than a straight angle; as angle *AOD*.



27. A **reflex angle** is an angle greater than a straight angle, but less than two straight angles; as angle *AOF*.

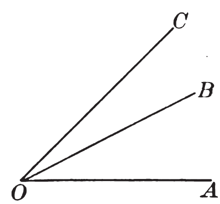


In this book, angles larger than a straight angle are not considered unless special mention is made of them.

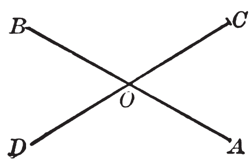
28. An **oblique angle** is an angle which is neither a right angle nor a straight angle. Hence, "oblique angle" is a general term for acute, obtuse, and reflex angles.

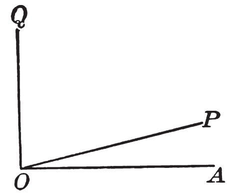
An **oblique line** is a line which makes an oblique angle with another given line.

29. **Adjacent angles** are angles which have a common vertex and a common side between them; as angles *AOB* and *BOC*.



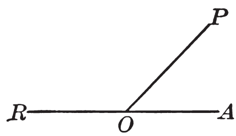
30. **Vertical angles** are angles which have a common vertex and the sides of one angle the prolongations of the sides of the other angle; as the angles *AOC* and *BOD*.





31. Complementary angles are two angles which together equal a right angle; as the angles AOP and POQ.

Hence, the **complement of an angle** is the difference between that angle and one right angle.



32. Supplementary angles are two angles which together equal two right angles (or a straight angle), as the angles AOP and POR.

Hence, the **supplement of an angle** is the difference between that angle and a straight angle.

EXERCISES: GROUP 5

Ex. 1. Draw an acute angle. An obtuse angle. A reflex angle.

Ex. 2. Draw two adjacent angles. Two vertical angles.

Ex. 3. How many degrees are there in the complement of an angle of 43°? In its supplement?

Ex. 4. Find the complement of 57° 19′. Of 62° 23′ 43″. Find the supplement of each of these angles.

Ex. 5. Draw an angle of 25°. What is the simplest way to construct the complement of this angle? The supplement?

Ex. 6. Is $\frac{5}{9}$ of a straight angle acute or obtuse?

Ex. 7. The dial of a clock is divided into sixty equal parts. On such a dial, how many degrees are there between two successive points of division?

Ex. 8. Find the number of degrees in the angle made by the hour hand and the minute hand of a clock at two o'clock. At three o'clock. At five o'clock.

Ex. 9. Find the number of degrees in the angle made by the hands of a clock at 1:30 o'clock. At 2:15. At 8:45.

Ex. 10. How long does it take the minute hand of a clock to turn through an angle of 60°? Of 50°? Of 240°? How long does it take the hour hand to turn through these angles?

Ex. 11. When a wheel makes $2\frac{1}{4}$ revolutions, through how many degrees does one of the spokes turn?